

Resonance

The Shocking But True Story of Stored
Energy and Opposing Phase,

or

How Capacitors and Inductors Learned
to Currently Cooperate.

Mechanical Resonance



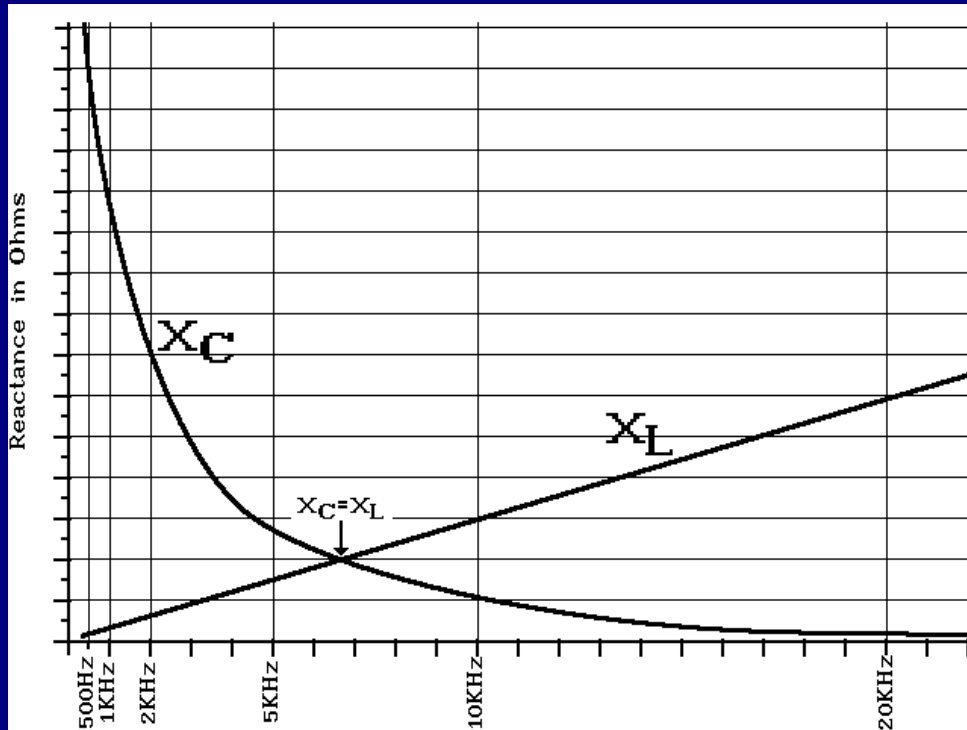
Resonant Systems share a few common traits.

Energy Input to the System is alternately stored and released.

Frequency of Resonance is usually fixed.

Energy is Dissipated over time due to internal or external losses.

The Resonant Frequency



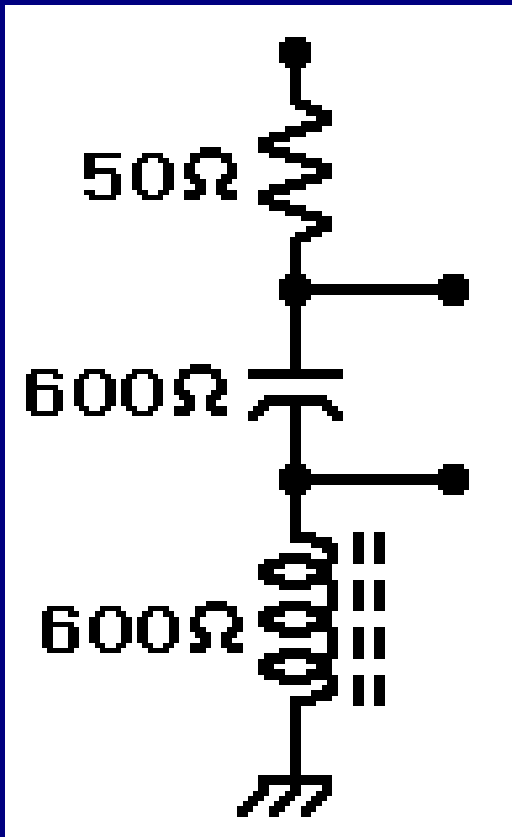
Many systems become resonant at some frequency.

In electronics, a capacitor and an inductor will become resonant when

$$X_C = X_L$$

This usually occurs at only one frequency.

A Series Coil and Capacitor



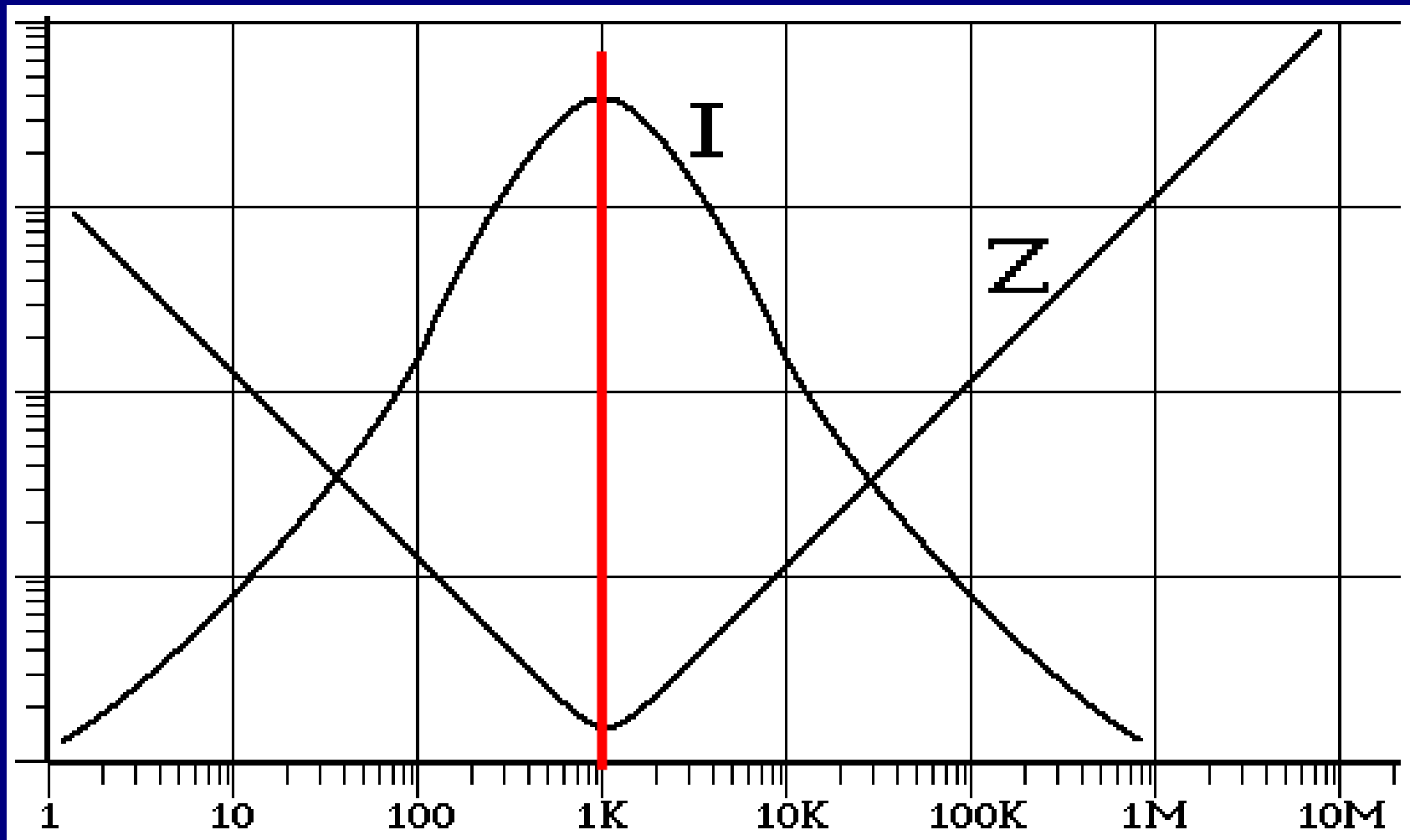
This circuit is resonant because $X_C = X_L$

Opposite Reactances Cancel, so the circuit appears only resistive.

If 10 ac volts at F_c was applied we would expect a current of $10V / 50\ \Omega = 200\text{mA}$.

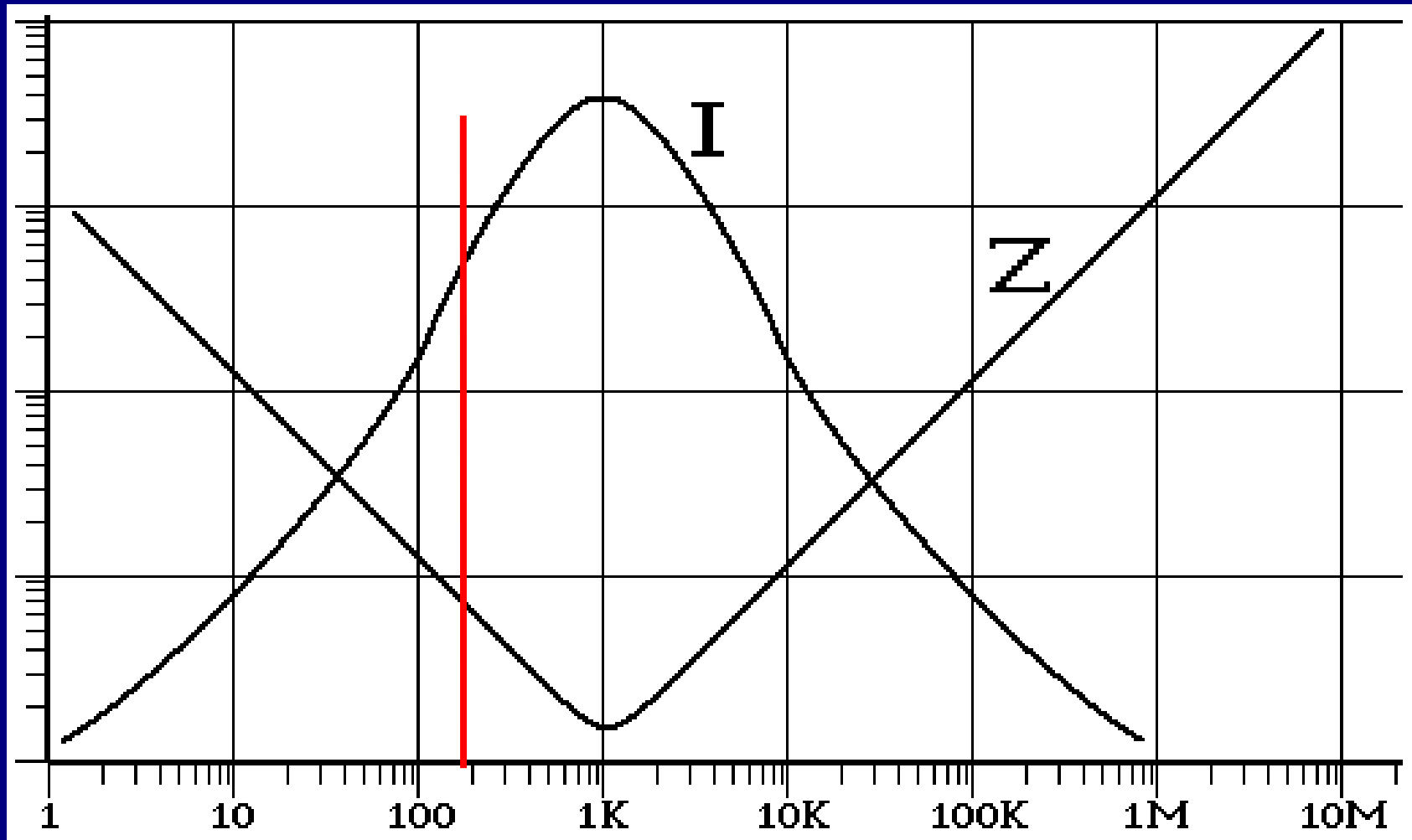
With real components the current will be somewhat less.

Impedance and Current vs Frequency



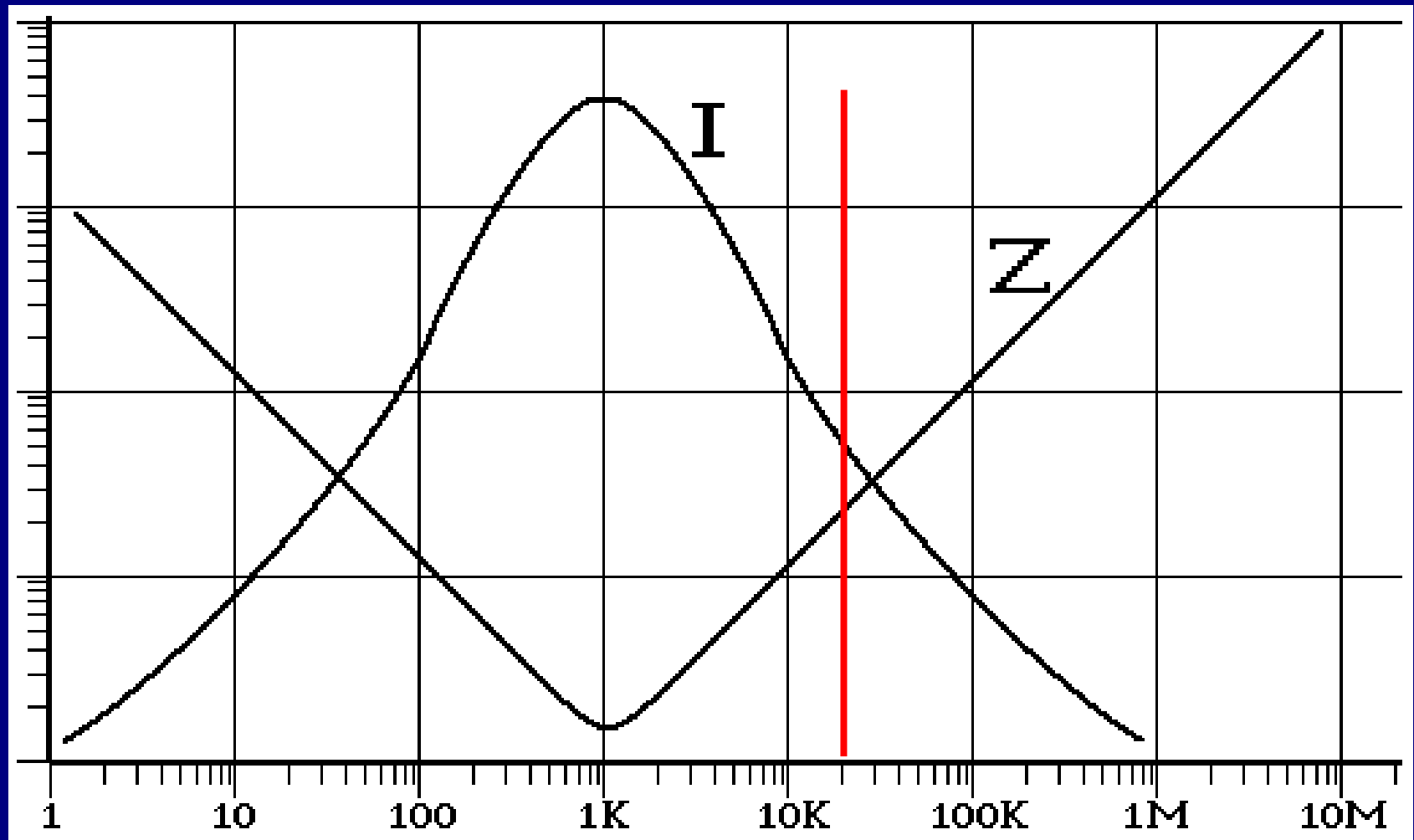
Current Peaks at Resonance, because the Impedance has reached its lowest value.

Impedance and Current vs Frequency



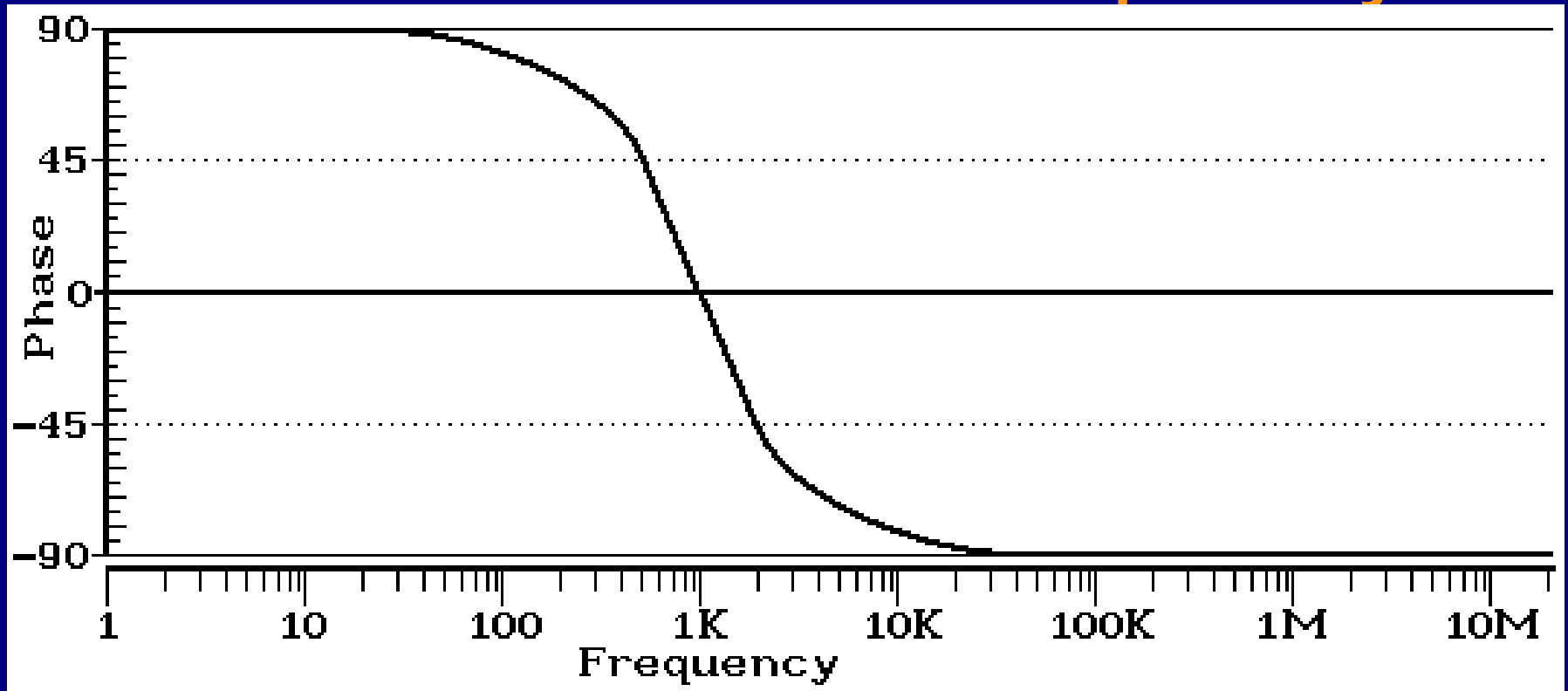
Below Resonance the Capacitor Reactance is high and climbing, limiting current.

Impedance and Current vs Frequency



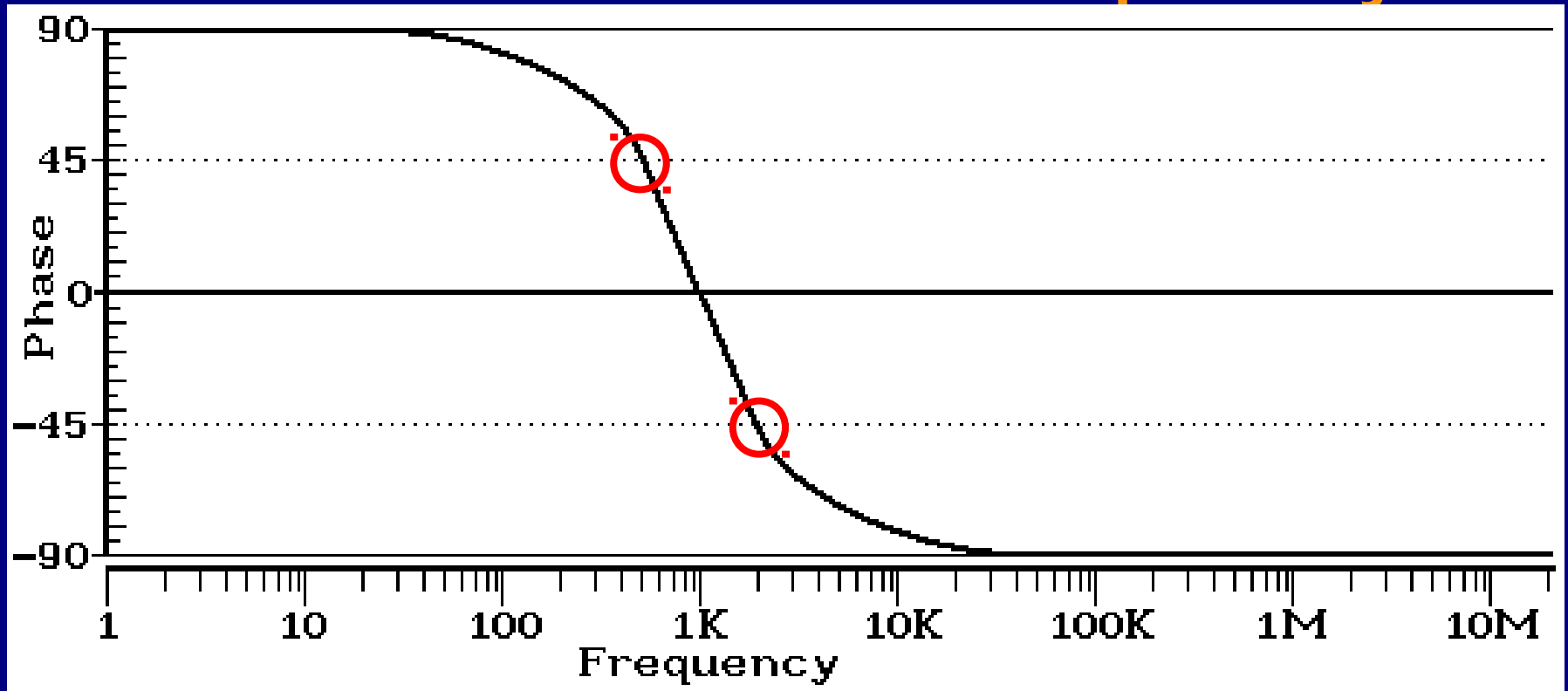
Above Resonance the Inductor Reactance is higher and climbing, limiting current.

Phase Shift with Frequency



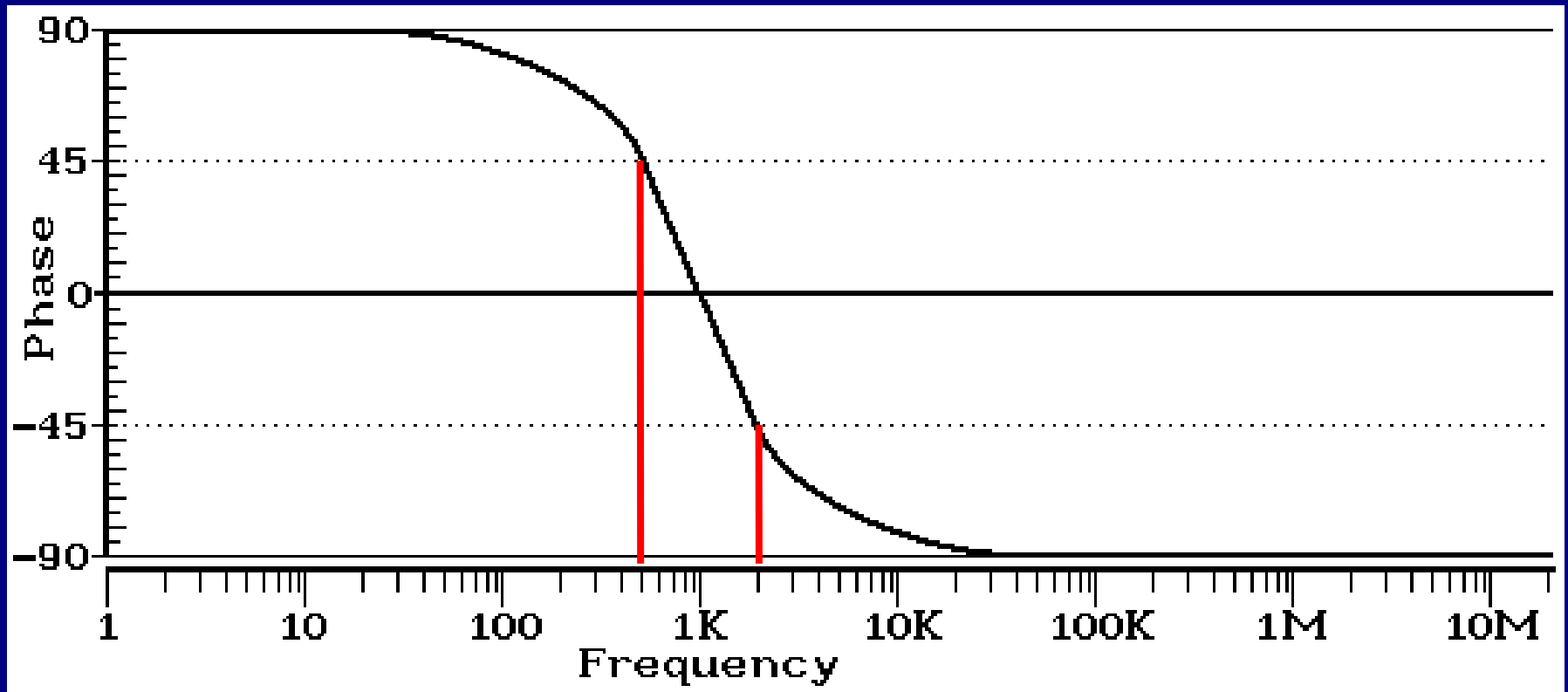
The phase shift of the series LCR circuit is capacitive (current leads voltage) at low frequency, through zero at resonance, and inductive (current lags voltage) at high frequency.

Phase Shift with Frequency



The -45° and $+45^\circ$ points are marked, because at that phase shift the reactance of the series circuit is equal to the circuit resistance. Half the circuit power is real, and half is reactive power.

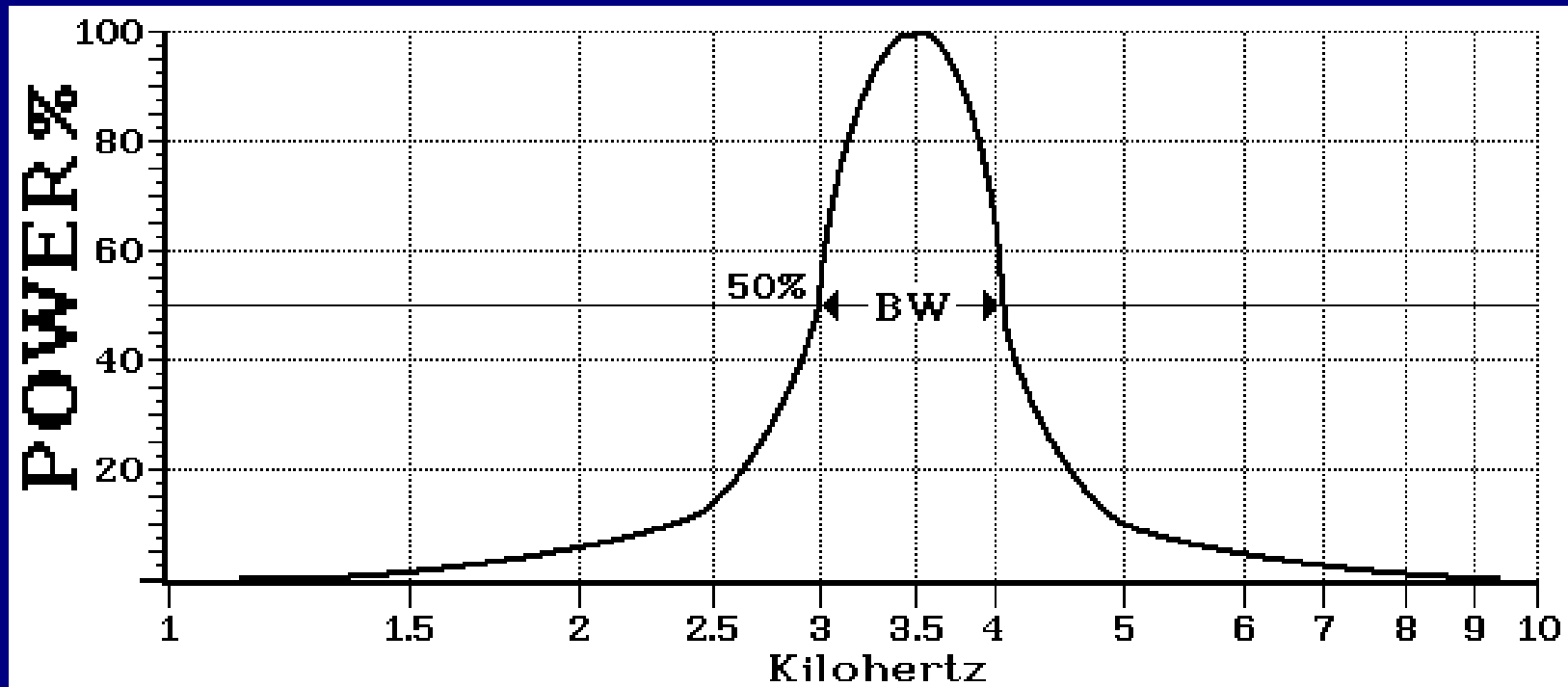
Phase Shift with Frequency



The frequency range between the -45° and $+45^\circ$ phase shift points is called the Bandwidth of the series circuit.

$$BW = F2 - F1$$

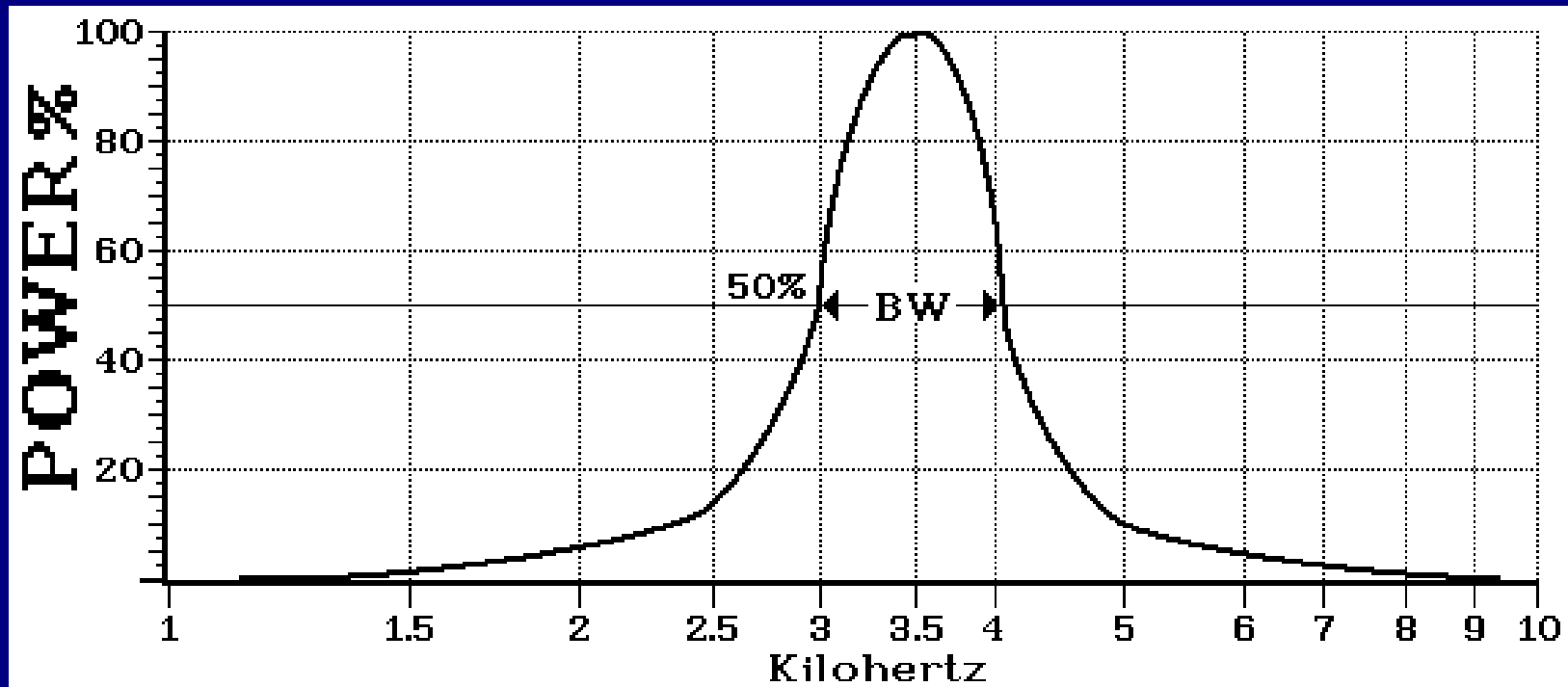
Power in a Series Circuit



Power is maximum at resonance, and drops off on either side.

The Half Power Point, -3dB points, also defines the circuit Bandwidth.

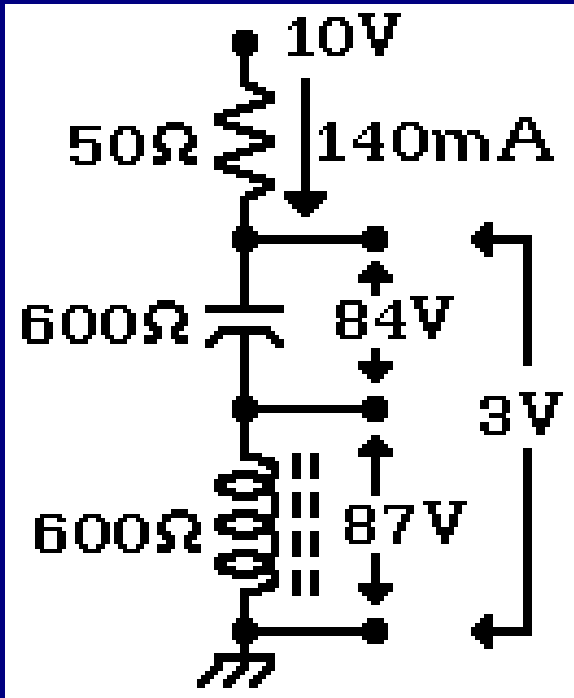
Power in a Series Circuit



The Bandwidth is equal to the Resonant Frequency divided by the circuit Q.

$$BW = \frac{F_R}{Q} = f_{HIGH} - f_{LOW}$$

A Real R-C-L Circuit



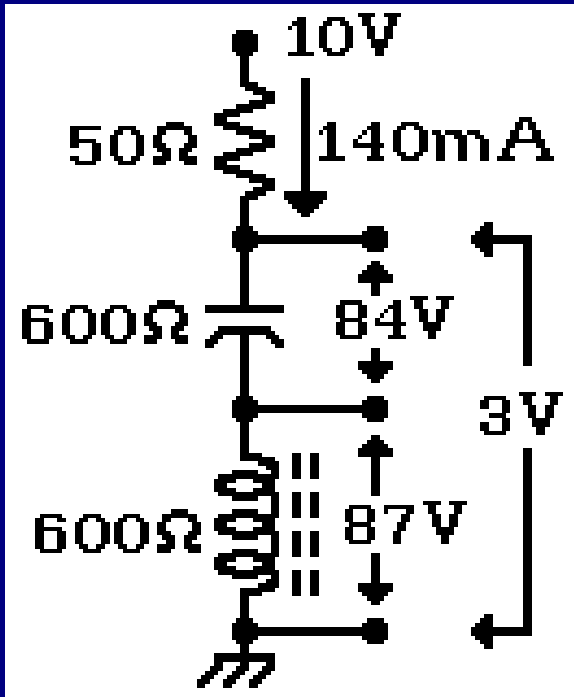
With real components we measure 140mA.

We measure 84 volts across the capacitor, the product of current and reactance.

We measure 87 volts across the coil, the product of current and reactance, and ?

The total circuit reactance Z is E / I . $10V / 140mA = 71.43\Omega$

A Real R-C-L Circuit



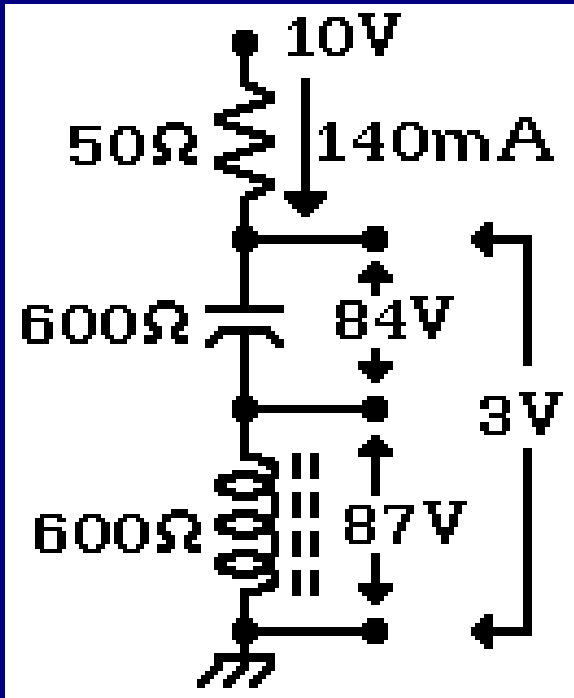
The circuit resistance is 71.4Ω
With 50Ω in the Resistor there must be 21.4Ω elsewhere.

The extra voltage across the coil shows the resistance must be in the coil.

The DC Multimeter measures the coil as 8Ω .

The difference is the AC Effective Resistance due to wire skin effect at Radio Frequencies.

A Real R-C-L Circuit



Resistance in a resonant circuit creates a loss of energy, and limits the maximum current flow.

The Ratio of Reactive Power to Real Power is a measure of the Quality of a resonant circuit.

$$Q = \frac{\text{Reactive Power}}{\text{Real Power}} = \frac{X_L}{R} = \frac{V_X}{E}$$

The Resonance Formula

At resonance:

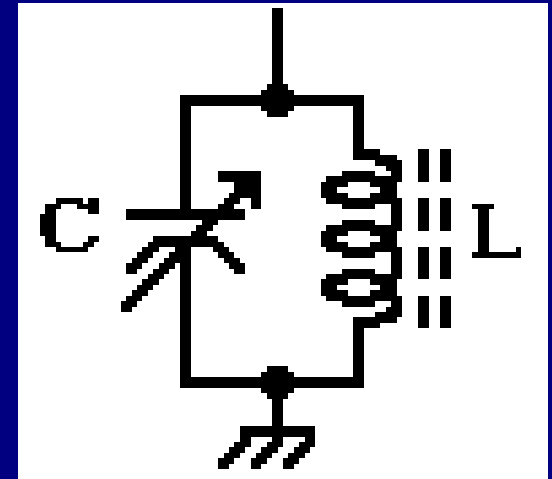
$$2\pi fL = \frac{1}{2\pi fC}$$

Rearrange to:

$$f^2 = \frac{1}{2^2 \pi^2 LC}$$

And simplify to:

$$f = \frac{1}{2\pi \sqrt{LC}}$$

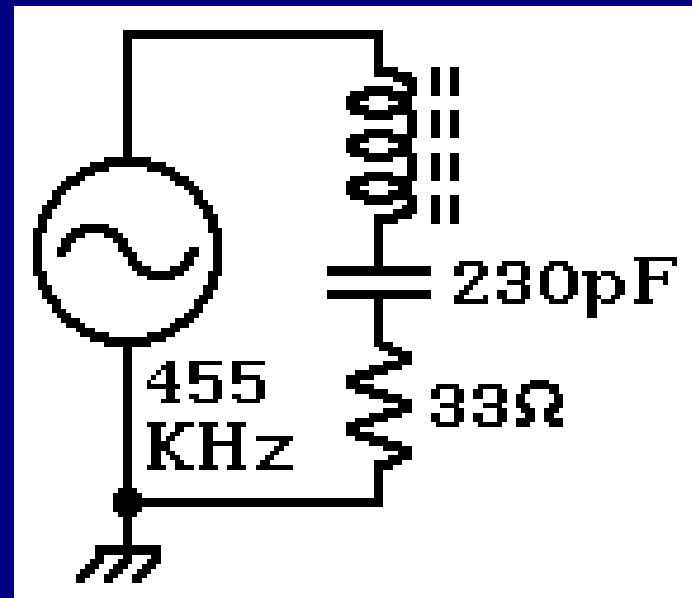


Given C and f, Find L

When designing a resonant circuit you may know the desired frequency and the circuit capacitance, but need to calculate the Inductance.

What is L?

$$L = \frac{\left(\frac{1}{2\pi f}\right)^2}{C}$$

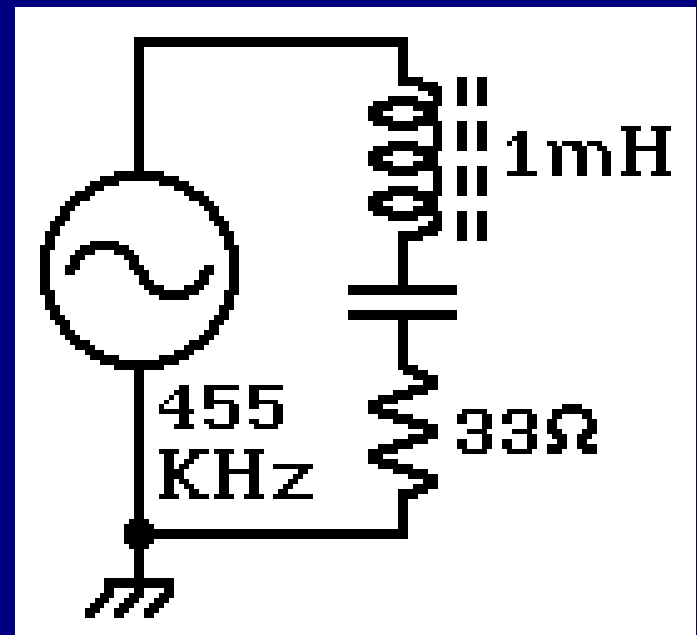


Given L and f, find C

You might have the Inductance and the frequency, and need capacitance.

$$C = \frac{\left(\frac{1}{2\pi f}\right)^2}{L}$$

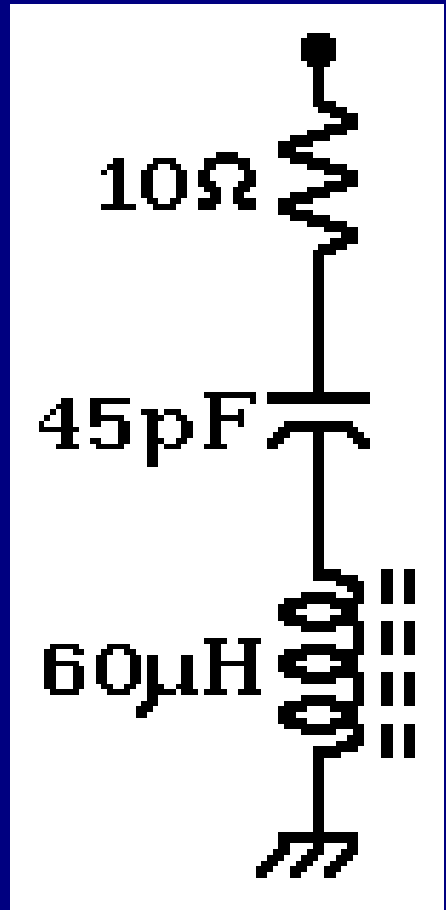
$$\frac{1.223538^{-13}}{1 \text{ mH}} = 122 \text{ pF}$$



Bandwidth

If the Resonant Frequency is 3.06MHz and the Q is 115.5, what is the Bandwidth?

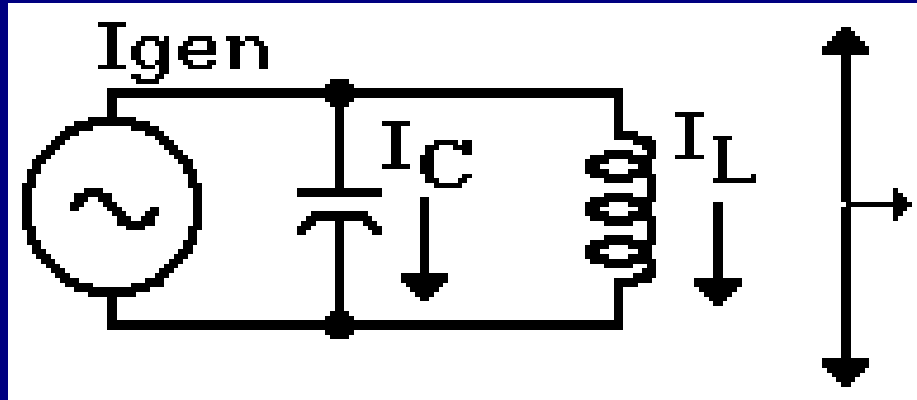
$$B_W = \frac{F_R}{Q} = \frac{3.06^6 \text{ Hz}}{115.5} = 26.52 \text{ KHz}$$



Parallel Resonance

Get “Tanked”

The Parallel Resonant Circuit



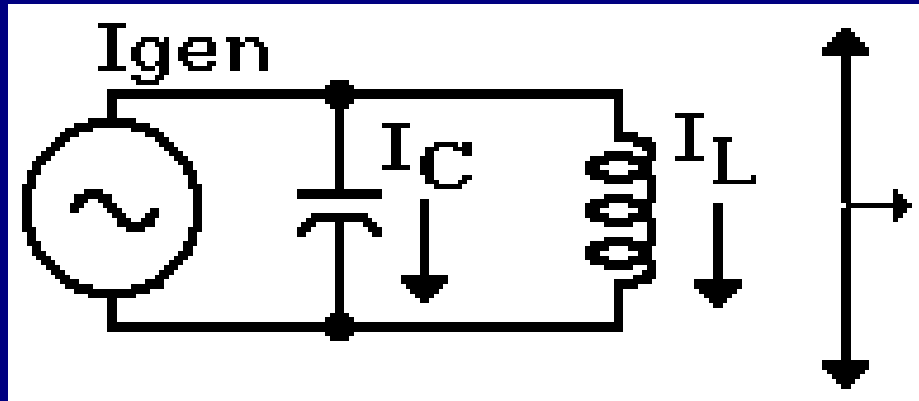
Voltage is common to the parallel LC circuit.

Current in the Capacitor, E / X_C , leads the applied voltage by 90° .

Current in the Inductor, E / X_L , lags the applied voltage by 90° .

Current in the Capacitor leads the current in the Inductor by 180° .

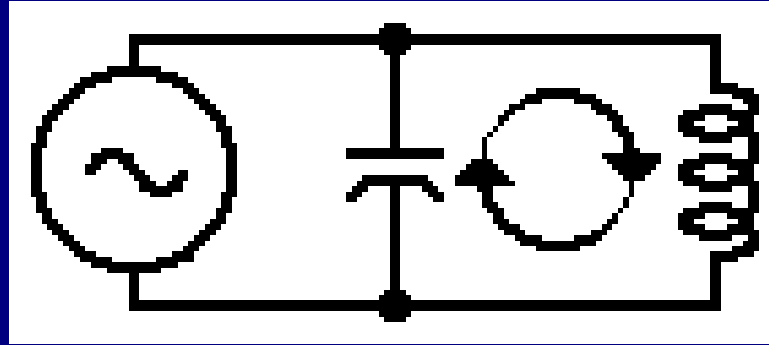
The Parallel Resonant Circuit



At Resonance, when $X_C = X_L$, the Capacitor current and the Inductor current totally cancel each other.

When the coil resistance is very low (High Q) there will be very little generator current, even though the reactive current is high.

The Parallel Resonant Circuit

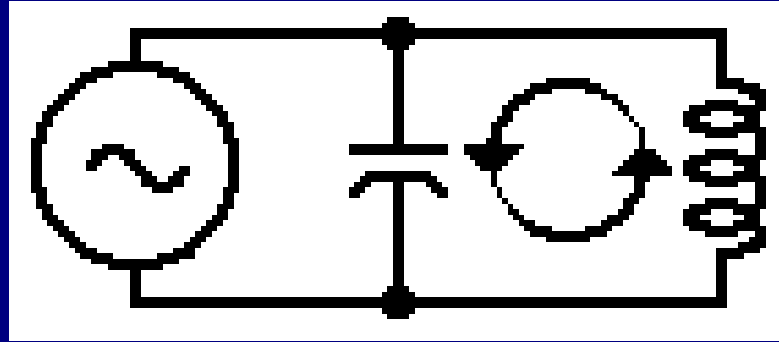


The current flows back and forth between the capacitor and the inductor.

The energy is alternately stored in the magnetic field of the coil, and the electrostatic field of the capacitor.

This “Tank” current = $Q \times$ the generator current.

The Parallel Resonant Circuit



The current flows back and forth between the capacitor and the inductor.

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This “Tank” current = $Q \times$ the generator current.

The Parallel Resonance Formula

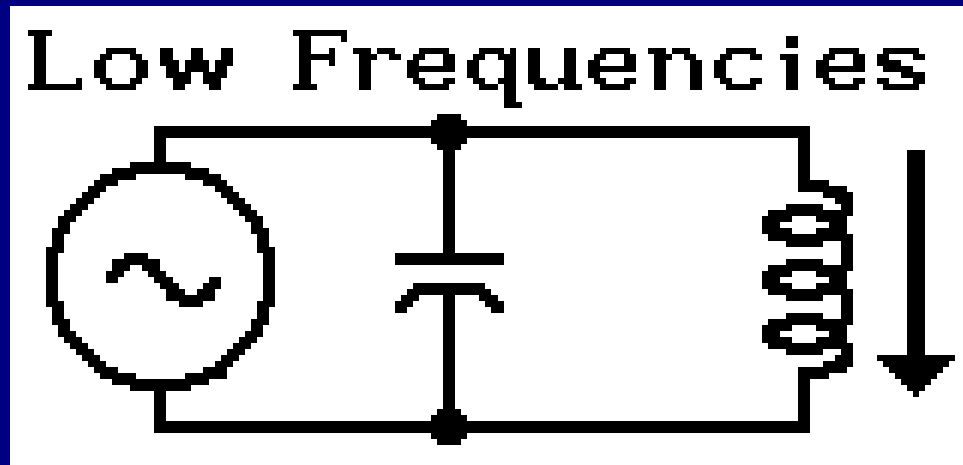
$$f = \frac{1}{2\pi\sqrt{LC}} \sqrt{1 - \frac{R_{\text{coil}}^2 C}{L}}$$

Where coil resistance R_L is within a factor of 100 of X_L .

In the more usual instance, when R_L is less than 1/100th of the coil reactance, we use:

$$f = \frac{1}{2\pi\sqrt{LC}}$$

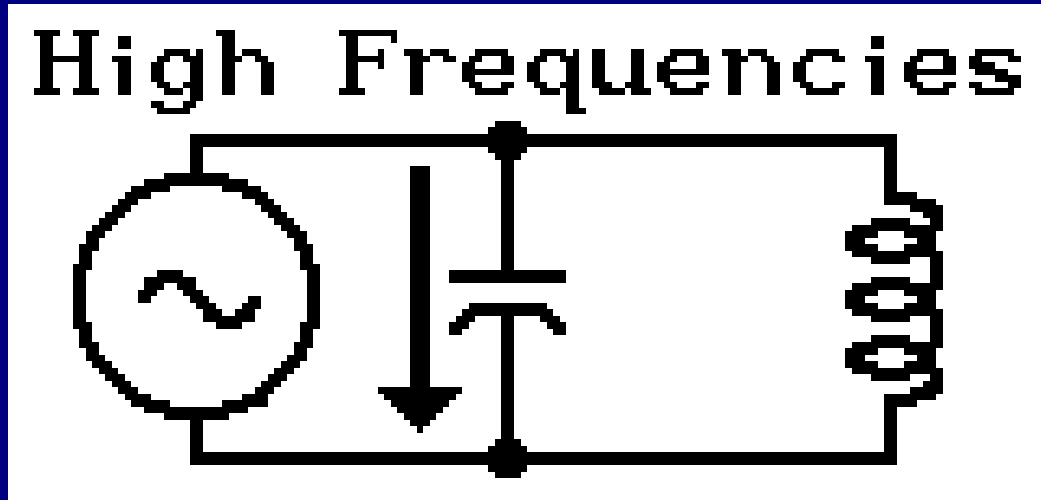
Off Resonance



Below Resonance the Inductive reactance is low and total circuit current is high.

The capacitive current is low, so the circuit current is inductive, with current lagging the applied voltage.

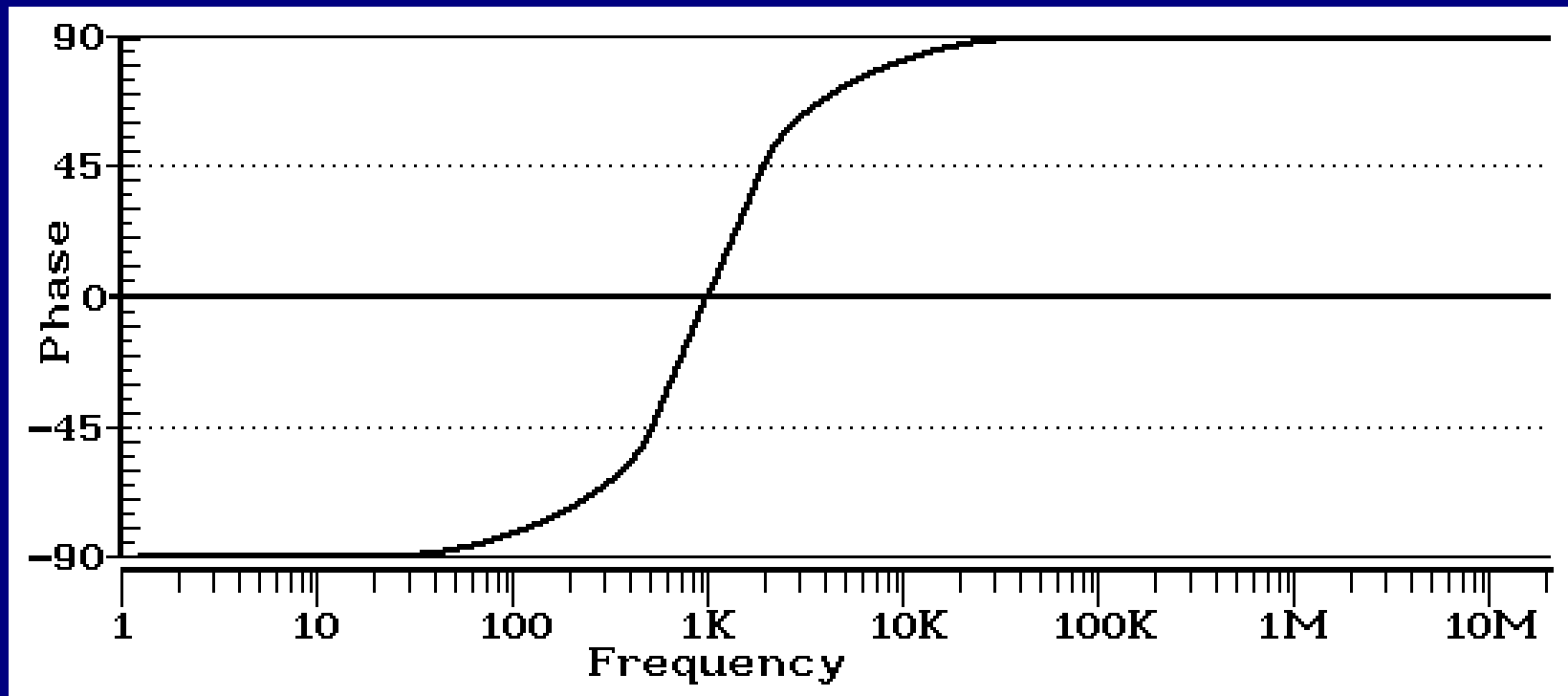
Off Resonance



Above Resonance the capacitor is low reactance and total circuit current is high.

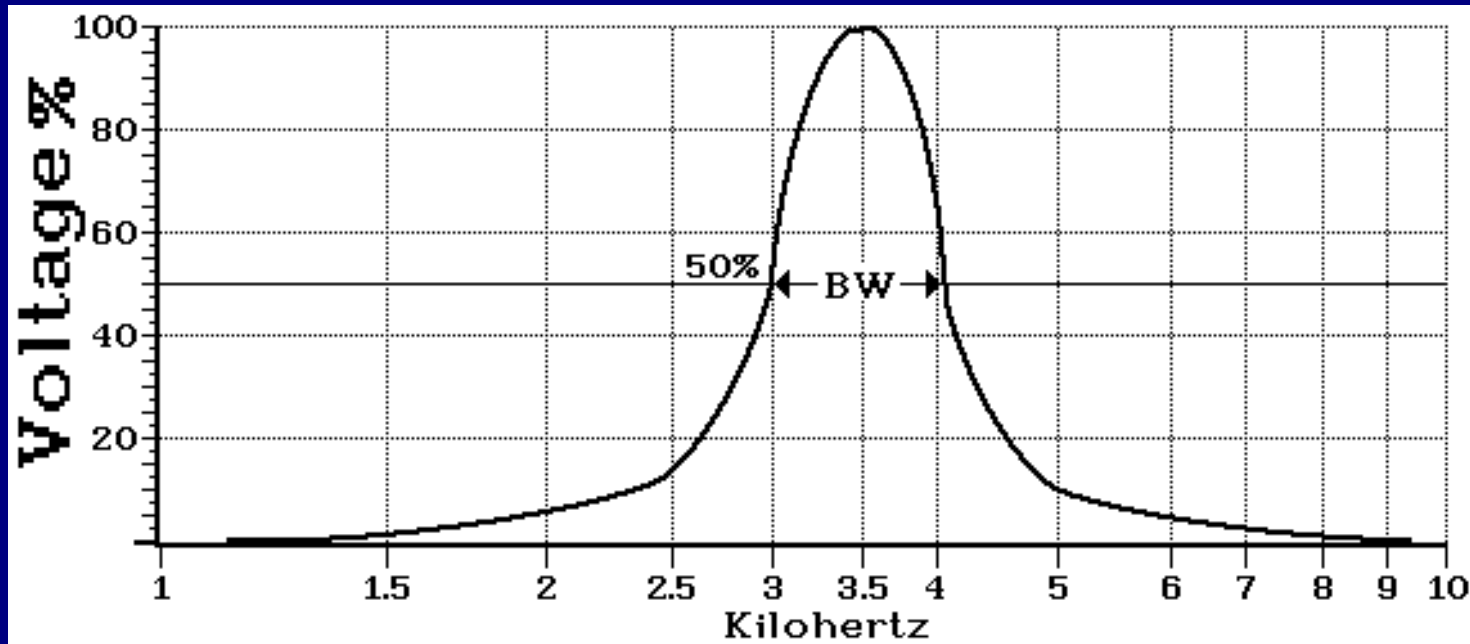
The inductive current is low, so the total circuit current is Capacitive, with current leading the applied voltage.

Parallel Current Phase



Below Resonance the current is Inductive, lagging the applied voltage, at Resonance the phase shift is zero, and Above Resonance the current is capacitive, leading the applied voltage.

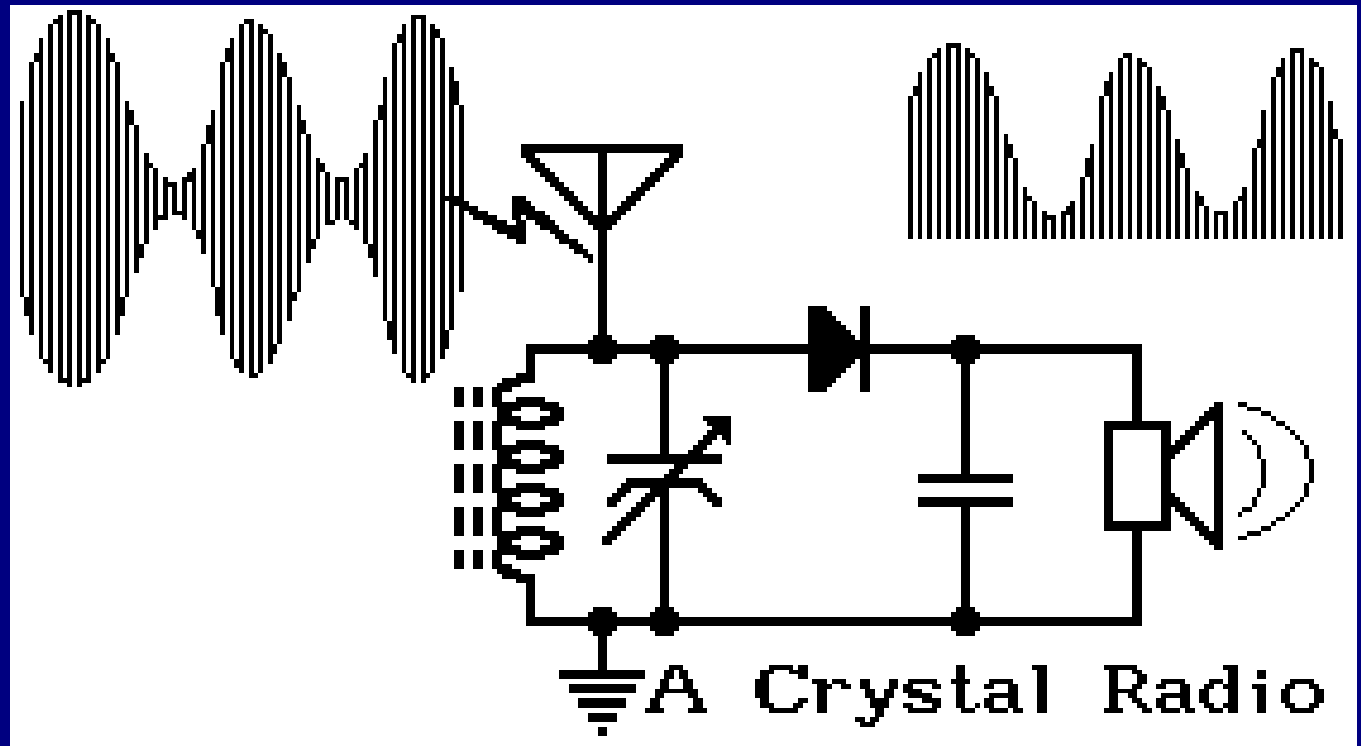
Parallel Selectivity Curve



The voltage across a parallel LC circuit is maximum at resonance, dropping off quite quickly on either side.

$$\text{Bandwidth} = Fr / Q = R_L / X_L$$

The Crystal Radio

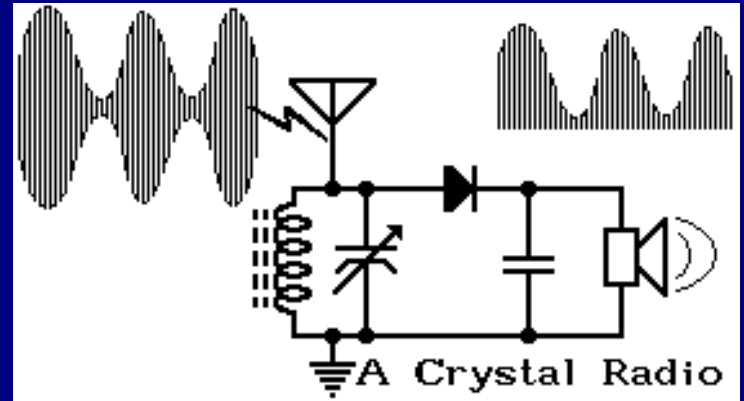


The high Q tank circuit increases the voltage of the tiny signal induced onto the antenna, but only at the resonant frequency.

The Ge or Si diode rectifies the radio signal.

High impedance earphones create sound.

The Crystal Radio

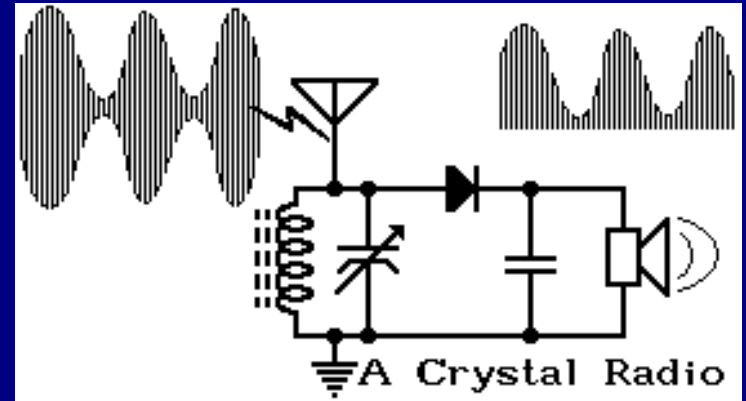


A variable capacitor is available with a range of 365pF to 35pF.

What coil value is required to receive 540KHz?

$$L = \frac{\left(\frac{1}{2\pi f}\right)^2}{C} = \frac{86.866^{-15}}{365^{-12} F} = 238 \mu H$$

The Crystal Radio

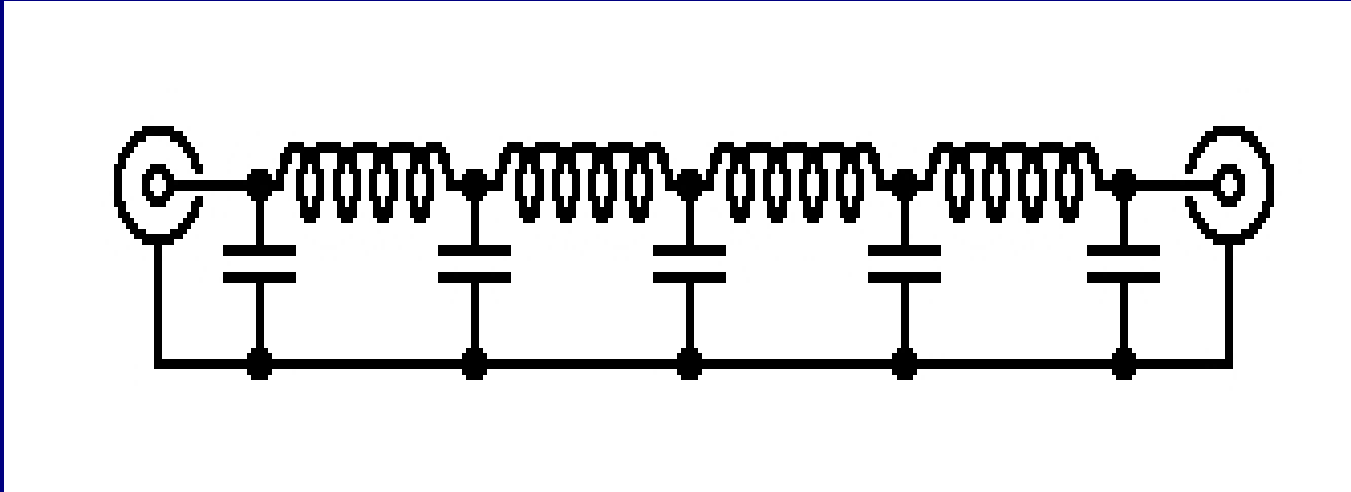


When the variable capacitor is adjusted to the minimum 35pF, what frequency will we receive with our fixed coil value ?

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{35^{-12} F 238^{-6} H}} = 1.7438 \text{ MHz}$$

This is slightly higher than the top of the AM Radio band.

A Low Pass Filter



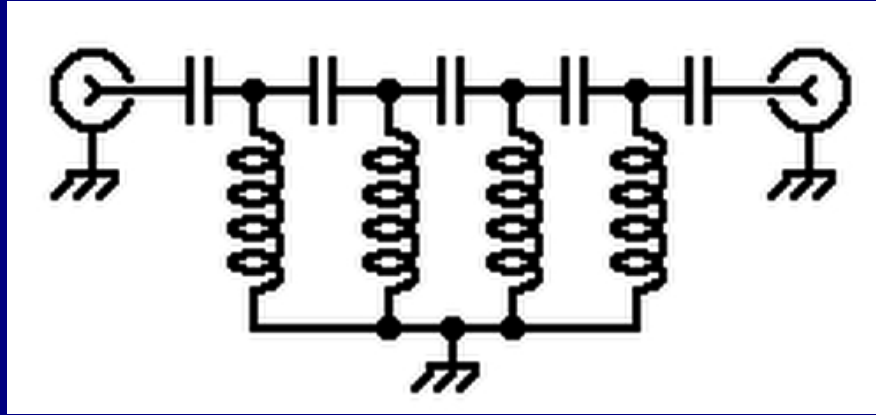
How do you know if this Filter is Low Pass?

Will the filter pass DC from Input to Output? If so, the filter will also likely pass low frequencies.

At some frequency the Inductive Reactance of the coils begins to block higher frequencies, while the lower capacitive reactances begin to bypass the signal to ground.

The Frequency where only Half of the Input Power makes it to the Output is known as the Cutoff Frequency.

A High Pass Filter

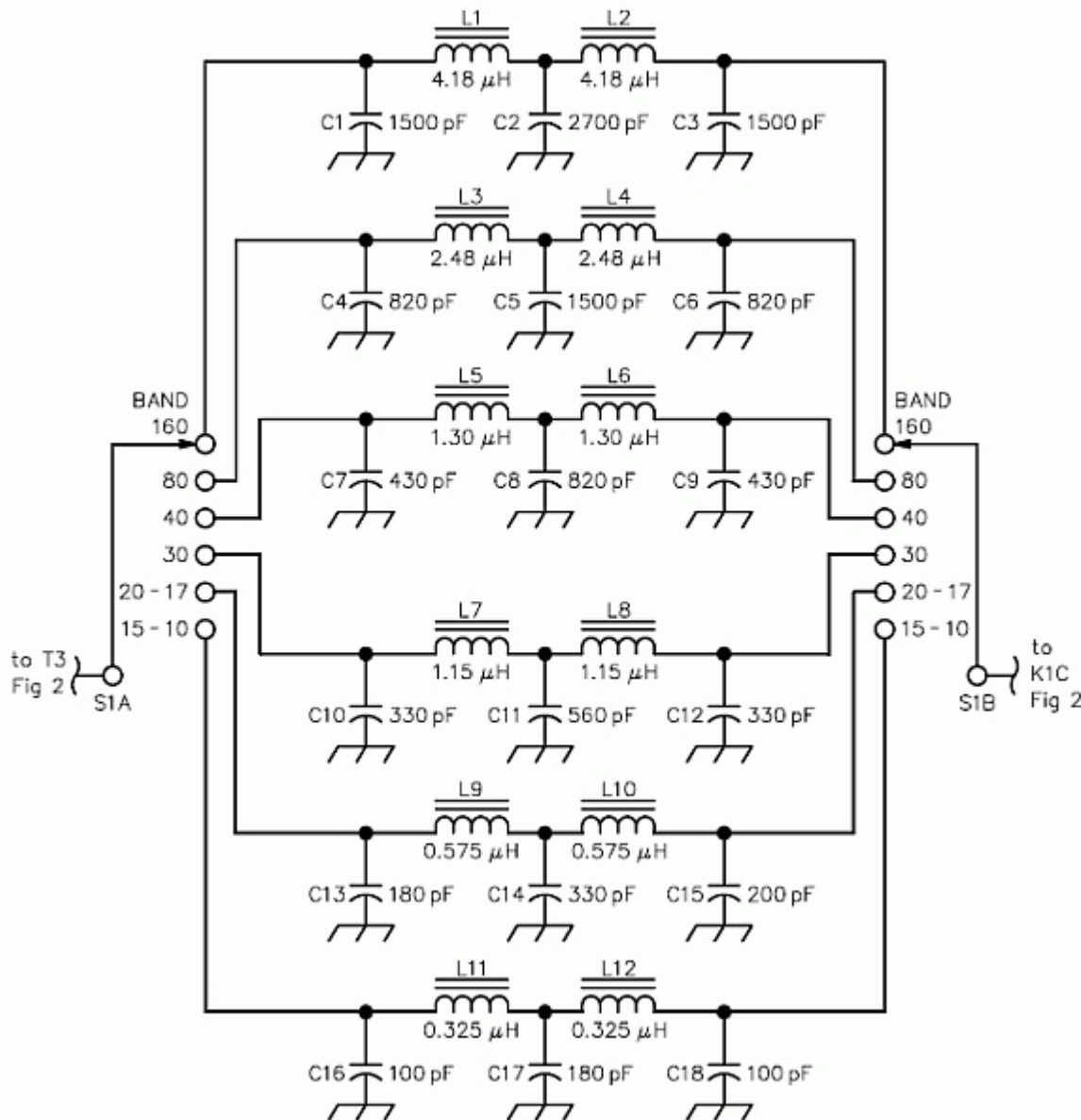


DC Current flow is blocked by the capacitors, so this filter is not a Low Pass Filter.

Signals at a high Frequency will see each Capacitor as almost a dead short circuit, while the Inductors will look almost like open circuits, so High Frequency Signals will easily flow from Input to Output. A High Pass Filter!

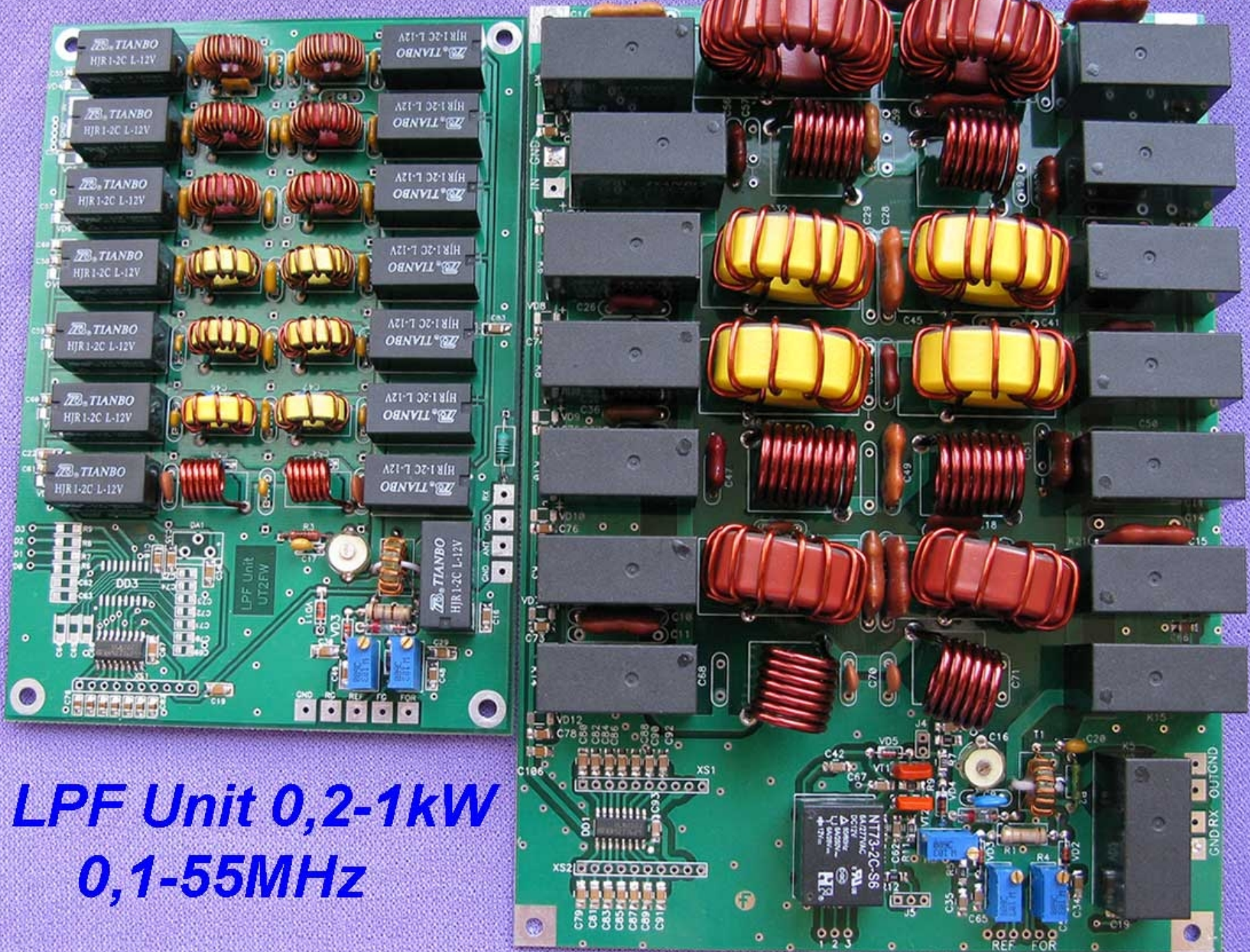
At some Lower Frequency the combination of High Capacitive Reactance that blocks signals and Low Inductive Reactance that bypass signals to ground will prevent signals from passing.

A Harmonic Blocking Filter



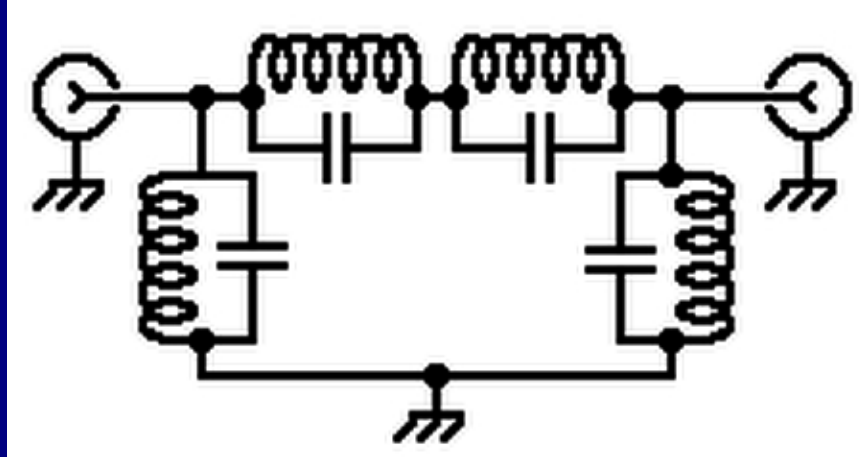
Low Pass Filters such as this are used in Transmitters to remove Harmonics which are produced by the RF Power Amplifiers so that harmful interference is not radiated.

A single filter will work well for two or three neighbouring bands at the higher frequencies.



**LPF Unit 0,2-1kW
0,1-55MHz**

Band Pass Filters

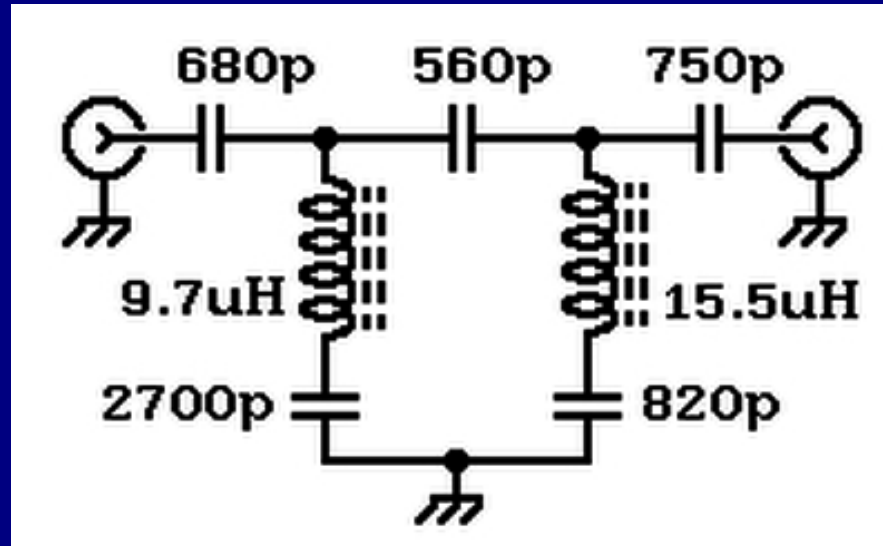


Band Pass Filters pass a narrow range of frequencies while blocking adjacent frequencies.

For example, a 40m BPF will be transparent from 7MHz~7.3MHz while preventing interaction with stations on 80m and 20m.

Band Pass Filters allow Multiple Stations to operate together on different bands for Field Day and Contests with minimal interference.

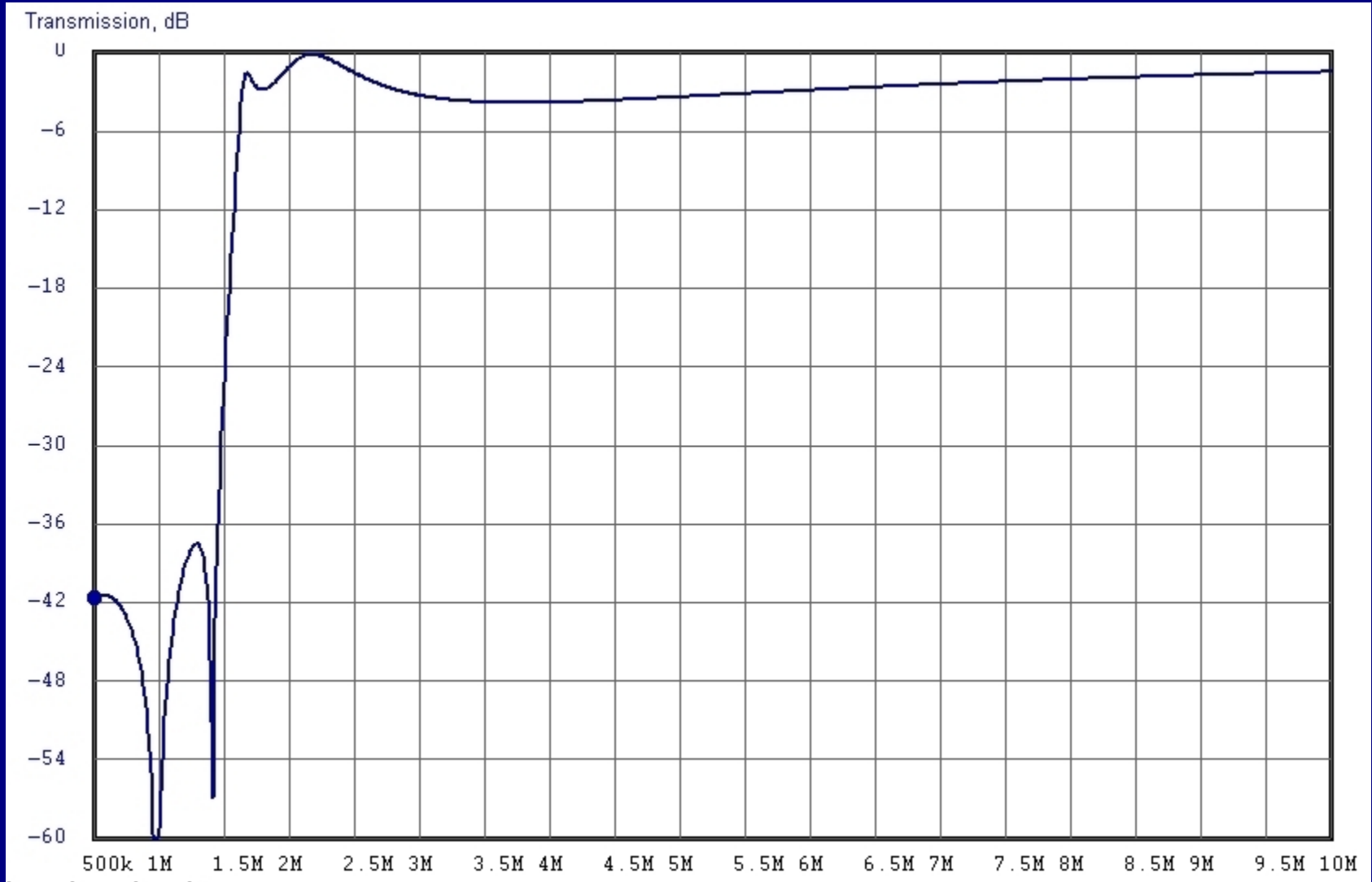
A Broadcast Band Reject Filter



This is a High Pass Filter with two Frequency Rejecting Series Resonant circuits. One is tuned to 980kHz to remove CFPL and the other is tuned to 1290kHz to remove CKSL. Both are strong signals that can overload amateur receivers.

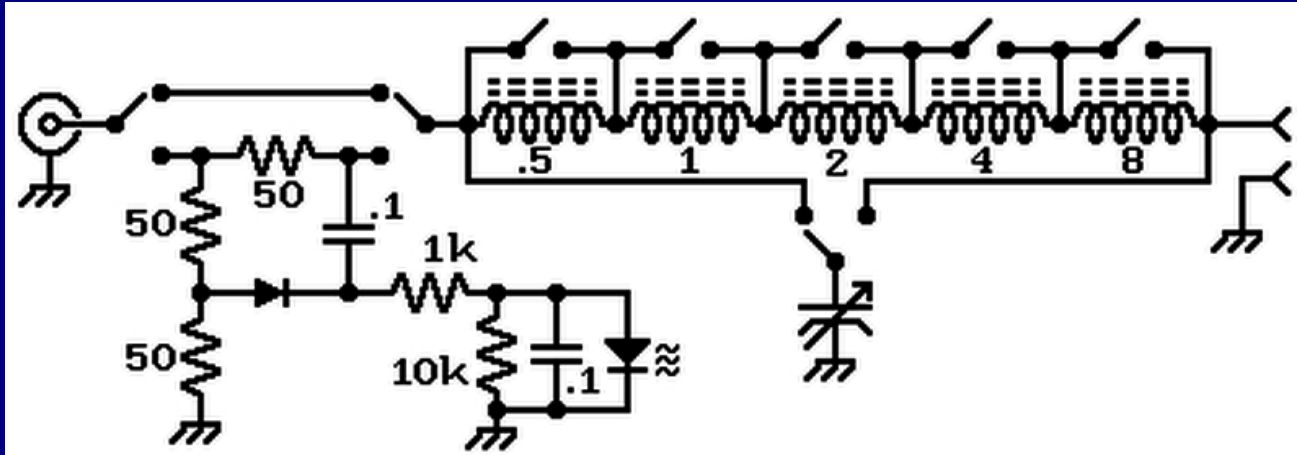
Above the Broadcast Band the filter begins to pass signals, and by the 160m amateur band, 1.8MHz the signals are passed with low loss.

Attenuation Curve for the Broadcast Band Reject Filter



Note the two deep reject notches for 980kHz and 1290kHz.

Impedance Matching With Coils and Capacitors



Inductors and Capacitors are used to form Impedance Matching Networks, also called “Antenna Tuners”.

The circuit shown above is a Low Pass C-L network, matching Low Z to 50Ω , or a L-C network, matching High Z to 50Ω .

A string of inductors, with values of $0.5\mu\text{H}$, $1\mu\text{H}$, $2\mu\text{H}$, $4\mu\text{H}$ and $8\mu\text{H}$, are shorted out if not needed for a match.

The Resistive/Diode Bridge at left measures the impedance match to 50Ω , and shows a low SWR when the LED dims.